1- Design of two way slab:
   a. ACI Direct Design Method.
   b. Equivalent Frame Method.
   c. Yield Line Theory.
2- Deflection and control of two way slab.
3- Shear of two way slab.
4- Prestressed of simply supported R.C beams.

- **References:**

- **Two way slab method of analysis:**
     a. Slab supported by brick wall.
     b. Slab supported by concrete wall.
     a. Slab supported by brick wall.
     b. Slab supported by concrete wall.
     c. Slab supported by steel beam.
  4. ACI Direct Design Method:
     It is a semi empirical procedure for slab analysis and it is application:
     a. Slab supported by beams or walls.
     b. Flat slab system.
     c. Flat plate slabs system.
     d. Two way grid slabs system.
5. Equivalent Frame Method:

It is an approximate elastic analysis of slab system and applied to:

a. Slab supported by beam or wall.
b. Flat slabs.
c. Flat plate slab.
d. Two way grid slab analysis.

6. Yield Line Theory:

It is a method of slab based on elastic considerations and the condition obtained in the structures just prior to failure. The method is usually used to analysis slab of irregular as well as regular slabs for various loading and support condition.

1. ACI – Direct Design Method:

It is an approximate semi empirical method for analysis two way slab system it's applied for:

1. Slab supported by beam or wall.
2. Flat slabs System.

3. Flat plate slab system.

4. Two way grid slab analysis (waffle slab or two way ribbed slab).

- **Limitations D.D.M:**
  1. There shall be a minimum of three continuous spans in each direction.
  2. Panels shall be rectangular with a ratio of longer span to shorter span center to center of supports with in a panel not greater than 2.
  3. Successive span lengths center to center of supports in each direction shall not differ by more than one-third the longer span.
  4. Column may be offset a maximum of 10% of span in the direction of offset from either axis between center lines of successive columns.
  5. Loads must be due to gravity only and live load must not exceed 2 times the load (L.L. / D.L. ≤ 2).

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6. For a panel with beams between supports on all sides, the relative stiffness of beams in two perpendicular direction given by the ratio:

\[
0.2 \leq \frac{\alpha_1 L_2^2}{\alpha_2 L_1^2} \leq 5.0
\]

- **Main steps of D.D.M.**:
  1. Calculate the total factored static moment \( M_o \).
  2. Distribution of \( M_o \) into positive and negative moments.
  3. Distribution of each positive & negative moments into column & middle strip moments.

\[
M_o = \frac{1}{8} W_u L_2 L_n^2
\]

Where:

\[
W_u = 1.2 \text{ D.L.} + 1.6 \text{ L.L.}
\]

- Width of frame measured from center to center of panels for interior frame and from center of panel to edge of panels to edge of exterior frame.
- Clear span of \( l_1 \) measured from face to face of support column, column capital, bracket, wall.

**Note:**

\[ L_n \geq 0.65 L_1 \]

- **Positive and negative moments**:
  a. For interior span:

\[
M^+ = 0.35 M_o
\]

\[
M^- = 0.65 M_o
\]
b. For exterior span, go to the table of (13-6-3-3).

- Column strip moment:

\[ c.s\ (width) = 0.25(\frac{L_1}{2} + L_2) \]

\[ L_1 \]

\[ L_2 \]

(0.5 \( L_2 \wedge \frac{w}{2}) - c.s(\text{width})

- For interior frame:
  
  \[ C.S_{\text{total}} = (c.s_1 \text{ and } c.s_2) \]
  \[ M.S_{\text{total}} = M.S_{\text{int.fram}} + M.S_{\text{int.or.ext.fram}} \]

- For exterior frame:
  
  \[ C.S_{\text{total}} = C.S \]
  \[ M.S_{\text{total}} = M.S_{\text{int.fram}} + M.S_{\text{ext.fram}} \]
  
  a. For positive moment (+M):
     
     From table (13.6.4.1) in ACI code.
  
  b. For negative moment exterior (-\( M_{\text{ext}} \)):
     
     From table (13.6.4.2)

\[ \beta = \frac{c}{2I_s} \]

\[ I_s = \frac{L_t \ast L^3}{12} \]

\[ L_t = \text{length of torsional member} \]

\[ L_t \text{ for exterior strip} = L_2 \]

\[ L_t \text{ for interior strip} = \frac{1}{2} L_2 + \frac{1}{2} L_2 \]

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\[ C = \sum \left[ \left(1 - 0.63 \frac{x}{y}\right) \frac{x^3}{3} \right] \]

\( x = \) short direction .
\( y = \) long direction .

If no beam then " \( T_m \) " (torsion member):

1. \( c_1 \)

2. \( c_1 \)

3. \( b_f > c_1 \)

4. \( b_f < c_1 \)
Note: (13.6.4.3)
Negative moment are uniformly distributed cross $L_2$ when the support width is:
$$c_2 \geq \frac{3}{4} L_2$$

Note: (13.6.4)
$$M_{ms1} = M_{ms}$$
$$M_{cs} = 0$$

- **Moment in beams:**
  
  Moment in beams form:
  1. Slab moment.
  2. Beam self weight.
  3. Partition.

1. Moment in beam from slab weight = $0.85 M_{cs}$
   
   If $\alpha_1 = \frac{L_2}{L_1} \geq 1$

2. If $\alpha_1 = \frac{L_2}{L_1} = 0$ moment in beam from slab weight = 0
3- Moment in beam from slab weight between 0-0.85 of col. Strip moment.
   IF $0 < \frac{\alpha_1 L_2}{L_1} < 1$

- Moment of slab from col. Strip mom. = $M_{c,s} - M_{beam}$
- Moment of middle strip = mom. Slab in middle strip only.

• Summary procedure of D.D. M.
  1. find $W_u = 1.2D.L + 1.6L.L$
  2. find $M_o = \frac{1}{8}W_uL_2L_n^2$
  3. find $M-$ and $M+$
  4. find $M_{c,s}$ from = $(?) \times (M- \text{ or } M+)$
     Then find $M_{m,s}$ from = $(M- \text{ or } M+) - M_{c,s}$
  5. If beam found
     a- $M = 0.85M_{c,s}$ (If $\alpha_1 \frac{L_2}{L_1} > 1$)
     b- $M = (0 - 0.85)M_{c,s}$ (If $0 < \alpha_1 \frac{L_2}{L_1} < 1$)
  6. $M_{c,c,s} = M_{c,s} - M_{beam}$
     a- $M_{c,c,s} = M_{c,s}$ (if no beam found).
  7. $M_{m,s} = (M- \text{ or } M+) - M_{c,s}$
  8. To find moment per meter :-
     a- $M_{c,s} = \frac{M_{c,s}}{\text{width of } c.s}$
     b- $M_{m,s} = \frac{M_{c,s}}{\text{width of } m.s}$

H.W :- chose for the below roof if the D.D.M applied or not :-
Assume :-
- L.L = 2.5 $\frac{kn}{m^2}$
- Imposed D.L = 1 $\frac{kn}{m^2}$
- Concrete density = 2.4 $\frac{kn}{m^3}$
- Thick of slab = 150 mm
EX(1): for the slab system shown in fig. below find all moment in x direction? assuming:

concrete cover = 30 mm .
S.L.L = 2.728 \( \frac{kn}{m^2} \)
S.D.L.(including self weight ) = 4.82 \( \frac{kn}{m^2} \)

\( f_c = 21 \text{ mpa} \), \( f_y = 420 \text{ mpa} \)
Col. Dia. = (250×250) mm
Edge beam depth = 600 mm
Int. beam depth = 500 mm

\[
\begin{align*}
W_{ult} &= 1.2 \times D.L + 1.6 \times L.L \\
W_{ult} &= 1.2 \times 4.82 + 1.6 \times 2.728 = 10.15 \frac{kn}{m^2} \\
M_o &= \frac{1}{8} W_{ult} L_2^2 L_n = \frac{1}{8} \times 10.15 \times 6 \times (4 - 0.25)^2 = 107.1 \frac{kn}{m^2} \\
\frac{L_2}{L_1} &= \frac{6}{4} = 1.5 \\
\alpha_1 &= \frac{I_b}{I_s}
\end{align*}
\]

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\[ I_b = k \frac{b_h h^3}{12} \quad (k = 1.5 \text{ ext.}, k = 2 \text{ int.}) \]
\[ = 2 \times \frac{250 \times 150^3}{12} = 5.21 \times 10^9 \text{mm}^4 \]
\[ I_s = \frac{b h^3}{12} = \frac{600 \times 150^3}{12} = 1.69 \times 10^9 \text{mm}^4 \]
\[ \alpha_1 = \frac{5.21}{1.69} = 3.1 \]
\[ \therefore \alpha_1 \frac{L_2}{L_1} = 3.1 \times 1.5 = 4.63 > 1 \]
\[ \beta_c = \frac{c}{2I_s} \]
\[ c = \sum [(1 - 0.63 \times \frac{x}{y}) \frac{x^3 y}{3}] \]
\[ c_1 = \sum [(1 - 0.63 \times \frac{250}{450}) \frac{250^3 \times 450}{3} + c = \sum [(1 - 0.63 \times \frac{150}{700}) \frac{150^3 \times 700}{3} \]
\[ = 2.2 \times 10^9 \]
\[ c_2 = \sum [(1 - 0.63 \times \frac{150}{450}) \frac{150^3 \times 450}{3} + c = \sum [(1 - 0.63 \times \frac{250}{600}) \frac{250^3 \times 600}{3} \]
\[ = 2.7 \times 10^9 \]
\[ I_s = \frac{l_c h^3}{12} = 1.69 \times 10^9 \text{mm}^4 \]
\[ \beta = \frac{2.7 \times 10^9}{2 \times 1.69 \times 10^9} = 0.798 \]
\[ M_{\text{int.}} = 0.7 \times 107.1 = 74.97 \text{ kn.m} \]
\[ M_{\text{+}} = 0.57 \times 107.1 = 61.047 \text{ kn.m} \]
\[ M_{\text{ext.}} = 0.16 \times 107.1 = 17.136 \text{ kn.m} \]

- For \( M_{\text{int.}} \)
\[ M_{cs} = 0.6 \times 74.97 = 44.982 \text{ kn.m} \]
\[ M_{csbeam} = 0.85 \times 44.982 = 38.235 \text{ kn.m} \]
\[ M_{cs \text{ slab}} = 44.982 - 38.235 = 6.75 \text{ kn.m} \]
\[ M_{m.s} = 74.97 - 44.982 = 29.9 \text{ kn.m} \]
- For $M_+$
  
  \[
  M_{cs} = 0.6 \times 61.047 = 36.63 \text{ kn.m}
  \]
  
  \[
  M_{csbeam} = 0.85 \times 36.63 = 31.13 \text{ kn.m}
  \]
  
  \[
  M_{cs\text{ slab}} = 36.63 - 31.13 = 5.49 \text{ kn.m}
  \]
  
  \[
  M_{m,s} = 61.047 - 36.63 = 24.41 \text{ kn.m}
  \]

- For $M_{-\text{ext}}$
  
  \[
  M_{cs} = 0.8736 \times 17.13 = 14.96 \text{ kn.m}
  \]
  
  \[
  M_{csbeam} = 0.85 \times 14.96 = 2.29 \text{ kn.m}
  \]
  
  \[
  M_{cs\text{ slab}} = 14.96 - 2.29 = 12.67 \text{ kn.m}
  \]
  
  \[
  M_{m,s} = 17.13 - 14.96 = 2.17 \text{ kn.m}
  \]

**For frame B :-**

\[
L_2 = \frac{6}{2} + \frac{0.25}{2} = 3.125 \text{ m}
\]

\[
I_s = \frac{L_t h^3}{12} = \frac{3.125 \times 0.15^3}{12} = 8.789 \times 10^{-4} \text{ m}^4
\]

\[
I_b = k \frac{b_w H^3}{12} = 1.5 \times \frac{0.25 \times 0.6^3}{12} = 6.75 \times 10^{-3} \text{ m}
\]

\[\alpha_1 = 7.68\]

\[
M_e = \frac{1}{8} W_u L_2 L_n^2 = \frac{1}{8} \times 10.15 \times 3.125 \times (4 - 0.25)^2 = 55.76 \text{ kn m}^2
\]

- Interior negative mom. \[= 0.65 \times 55.76 = 36.24 \text{ kn m}^2\]
  
  \[
  M_{cs} = 0.6 \times 36.24 = 21.75 \text{ kn.m}
  \]
  
  \[
  M_{csbeam} = 0.85 \times 21.75 = 18.49 \text{ kn.m}
  \]
  
  \[
  M_{cs\text{ slab}} = 21.75 - 18.49 = 3.26 \text{ kn.m}
  \]
  
  \[
  M_{m,s} = 36.24 - 21.75 = 14.5 \text{ kn.m}
  \]

- Interior positive mom. \[= 0.35 \times 55.76 = 19.51 \text{ kn m}^2\]
  
  \[
  M_{cs} = 11.7 \text{ kn.m}
  \]
  
  \[
  M_{csbeam} = 9.95 \text{ kn.m}
  \]
  
  \[
  M_{cs\text{ slab}} = 1.75 \text{ kn.m}
  \]
  
  \[
  M_{m,s} = 7.81 \text{ kn.m}
  \]

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Table (1) moment at interior frame :

<table>
<thead>
<tr>
<th>Mom.</th>
<th>C.S.S</th>
<th>M.S.S</th>
<th>beam</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.29</td>
<td>5.49</td>
<td>6.75</td>
</tr>
</tbody>
</table>

Table (2) moment at exterior frame :

<table>
<thead>
<tr>
<th>Mom.</th>
<th>C.S.S</th>
<th>M.S.S</th>
<th>beam</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2.8</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Deflection requirement according to ACI code (9.5.3):

- **Slab with beam**: the min. thickness of slabs depends on $\alpha_{fm}$ as follows:

1. $\alpha_{fm} \leq 0.2 \quad$ go to (9.5.3.2)
2. $0.2 < \alpha_{fm} \leq 2$
   
   $$h_{min.} = \frac{L_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)} \geq 125 \text{ mm}$$

3. $\alpha_{fm} > 2$
   
   $$h_{min.} = \frac{L_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} \geq 90 \text{ mm}$$

   $$\alpha_{fm} = \frac{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}{4}$$

   $L_n = $ clear span on long direction.

   $$\beta = \frac{L_n}{s_n}$$

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**note:** at discontinuous edge and edge beam should provided with a stiffness ratio $\alpha$ not less than 0.8 ; or minimum thickness required of eq. 9.12 and 9.13 should increased by 10% ($h_{\text{min.}} = h \times 1.1$).

**Ex(1):** for the slab –beam system, check the thickness requirement due to deflection ? assuming :

a- all col. (300×300)mm
b- Edge beam (300×650)mm
c- Int. beam (300×550) mm
d- $l_{\text{slab}}=150$ mm
e- $f_y=420$ mpa

**Sol:**

$$\alpha_1 = \frac{I_b}{I_s}$$

$$I_b = k \frac{bh^3}{12}$$

- to find approximate $k$ then :

1. $k = 1 + 0.2 \left( \frac{b_o}{b_w} \right)$
   - if $0.2 < \frac{T}{h} < 0.5$
   - if $2.0 < \frac{b_o}{b_w} < 4.0$
2. Otherwise:

\[
k = \frac{1 + \left(\frac{b_e}{b_w - 1}\right) \left(\frac{T}{h}\right) \left[4 - 6 \left(\frac{T}{h}\right) + 4 \left(\frac{T}{h}\right)^2 + \left(\frac{b_e}{b_w} - 1\right) \left(\frac{T}{h}\right)^3\right]}{1 + \left(\frac{b_e}{b_w} - 1\right) \left(\frac{T}{h}\right)}
\]

- For panel A:

\[
\alpha_{B1} = \frac{I_{b1}}{I_s}
\]

\[
T = \frac{150}{650} = 0.23
\]

\[
b_e = 800
\]

\[
b_w = 300
\]

\[
\frac{b_e}{b_w} = 2.67
\]

\[
\therefore k = 1 + 0.2 \left(\frac{b_e}{b_w}\right) = 1 + 0.2 \times 2.67 = 1.534
\]

\[
l_{b1} = \frac{kbh^3}{12} = 1.534 \times \frac{300 \times 650^3}{12} = 1.05 \times 10^{10} \text{mm}^4
\]

\[
l_{s1} = \frac{Lth^3}{12} = \frac{3.15 \times 0.15^3}{12} = 8.85 \times 10^8 \text{mm}^4
\]

\[
\alpha_{B1} = \frac{l_{b1}}{l_{s1}} = \frac{1.05 \times 10^{10}}{8.85 \times 10^8} = 11.85 > 0.8 \text{ o.k}
\]

\[
\alpha_{B5} = \frac{l_{b5}}{l_{s5}}
\]

\[
l_{b5} = 1.05 \times 10^{10} \text{mm}^4
\]

\[
l_{s5} = \frac{2.15 \times 0.15^3}{12} = 6.05 \times 10^8 \text{mm}^4
\]

\[
\therefore \alpha_{B5} = \frac{1.05 \times 10^{10}}{6.05 \times 10^8} = 17.36 > 0.8 \text{ o.k}
\]
\[ \alpha_{B7} = \frac{l_{b7}}{l_{s7}} \]

\[ T = \frac{150}{550} = 0.273 \]

\[ \frac{b_e}{b_w} = \frac{1100}{300} = 3.67 \]

\[ k = 1 + 0.2 \left( \frac{b_e}{b_w} \right) = 1 + 0.2 \times 3.67 = 1.733 \]

\[ l_{b7} = k \frac{bh^3}{12} = 1.733 \times \frac{300 \times 550^3}{12} = 7.21 \times 10^9 \text{mm}^4 \]

\[ l_{s7} = \frac{L_t h^3}{12} = \frac{4 \times 0.15^3}{12} = 1.125 \times 10^9 \text{mm}^4 \]

\[ \alpha_{B7} = \frac{7.21 \times 10^9}{1.125 \times 10^9} = 6.41 \]

\[ \alpha_{B3} = \frac{l_{b3}}{l_{s3}}, \quad l_{b3} = l_{b7} \]

\[ l_{s3} = \frac{L_t h^3}{7.21} = \frac{6 \times 0.15^3}{12} = 1.6875 \times 10^9 \text{mm}^4 \]

\[ \alpha_{B3} = \frac{1.6875}{4} = 4.273 = \alpha_{B4} \]

\[ \alpha_{fm} = \frac{11.85 + 17.36 + 6.41 + 4.273}{4} = 9.973 > 2 \]

\[ \therefore \text{use } h_{min.} = \frac{L_n \left( 0.8 + \frac{f_y}{1400} \right)}{36 + 9\beta} \geq 90 \text{ mm} \]

\[ \beta = \frac{l_n}{s_n} = \frac{5700}{3700} = 1.541 \]

\[ h_{min.} = \frac{5700 \left( 0.8 + \frac{420}{1400} \right)}{36 + 9 \times 1.541} = 125.74 \text{mm} > 90 \text{ o.k} \]

- \textbf{for panel B :}

\[ \alpha_{B2} = \alpha_{B1} = 11.88 \]

\[ \alpha_{B7} = \alpha_{B8} = \alpha_{B9} = \alpha_{B10} = 6.35 \]

\[ \alpha_{B4} = \alpha_{B3} = \alpha_{B11} = \alpha_{B12} = 4.23 \]

\[ 2 \alpha_{B7} + \alpha_{B4} + \alpha_{B2} = 7.18 > 2 \]

\[ h_{min.} = 125.7 \text{ mm} \]
- for panel C :-
  \( \alpha_{B6}=\alpha_{B5} = 17.55 \)
  \( \alpha_{B8}=\alpha_{B7} = 6.35 \)
  \( \alpha_{fm} = 8.62 > 2 \)
  \( h_{min.} = 125.7 \text{ mm} \)

- for panel D :-
  \( \alpha_{B4}=\alpha_{B12} = 4.23 \)
  \( \alpha_{B8}=\alpha_{B10} = 6.35 \)
  \( \alpha_{fm} = 5.29 > 2 \)
  \( h_{min.} = 125.7 \text{ mm} \)

Use large \( h_{min.} = 125.7 \text{ mm} \)
\( \therefore \) the slab thickness =150 mm is satisfactory \( T=150>125.7 \text{ o.k} \)

Shear Design of Slab :
- slabs with beams :
  a- one way shear (11.1 -11.5) (Beam Shear )

\[
\varnothing V_n \geq V_u
\]
\[
V_u \leq \varnothing V_c \text{ no need shear reinforcement}
\]
\[
V_u > \varnothing V_c \text{ need shear reinforcement}
\]
\[ V_u = W_u \cdot (x \cdot y) \]
\[ x = \frac{x^2}{2} - d - \frac{c_1}{2} \]
\[ \phi V_c = \phi 0.17 \sqrt{f_c} \cdot bd \]
\[ \phi (V_c + V_s) = V_u \]
\[ V_s = \frac{V_u}{\phi} - V_c \]

**b- two way shear action (punching shear):**

\[ V_u = W_u \left[(x \cdot y) - (x_1 \cdot y_1)\right] \]

To find \( V_c \) (11.12.2.1)

\[ V_c = 0.17 \left(1 - \frac{2}{\beta}\right) \sqrt{f_c} \cdot b \cdot d \]
\[ V_c = 0.083 \left(\frac{\alpha \cdot d}{b_0} + 2\right) \sqrt{f_c} \cdot b \cdot d \]
\[ V_c = 0.33 \sqrt{f_c} \cdot b \cdot d \]

\[ \beta = \frac{L}{s} \]

\( \alpha = 40 \) int. col.
\( \alpha = 30 \) edge col.
\( \alpha = 20 \) corner col.

**EX(1):** For typical interior panel of the flat plate slab system shown, determine the shear strength of slab at support with the allowable one? Increase the shear strength by reinforcement if it's not adequate?

Assuming:

\[ W_u = 16.5 \text{ kN/m}^2 \]
\[ f_c = 20 \text{ mpa} \]
\[ f_y = 400 \text{ mpa} \]

Col. Size \( = (300 \times 300) \text{ mm} \)
\[ d_{av.} = 200 \text{ mm}, T = 240 \text{ mm} \]

**Sol:**

1- **One way shear**

\[ V_u = W_u \times (x \times y) \]

\[ = 16.5 \left( 6600 \times \left( 3300 - 200 - \frac{300}{2} \right) \right) = 321.255 \text{ kn} \]

\[ \phi V_c = 0.17 \sqrt{f_{c'} bd} = 752.66 > V_u \text{ no need reinforcement} \]

2- **Two way shear:**

\[ V_u = W_u \left[ (x \times y) - (x_1 \times y_1) \right] \]

\[ V_u = 16.5 \left[ (6600 \times 6600) - (500 \times 500) \right] = 714.6 \text{ kn} \]

\[ V_c = 0.17 \left( 1 - \frac{2}{\beta} \right) \sqrt{f_{c'} bd} \]

\[ V_c = 0.083 \left( \frac{\alpha s d}{b_d} + 2 \right) \sqrt{f_{c'} bd} \]

\[ V_c = 0.33 \sqrt{f_{c'} bd} \]

\[ \phi V_c = 0.75 \times 0.33 \sqrt{20 \times 2000} \times \frac{200}{1000} = 442.74 \text{ kn} < V_u \text{ need rei.} \]
2- Equivalent Frame Method :

Step (1):
Equivalent slab beam section.
Transfer slab with beam in to equivalent plate.
\[ T_s < T_{equ.} < h \]

Step (2):
Equivalent columns.

Equivalent frame method steps :-
1- there is no limit or boundary for the E.F.M.
2- the three dimensional frame can be transform to the more than one frame of two dimensional frame.
3- the frame of 3D to be solve it can solve by cutting the section parallel the center line of columns.
4- If we have only dead and live loads we can tales one story and analysis it.
5- If the building exposed to lateral loads (wind, blast, or seismic load) we must analyzed the overall frame.
6- The equivalent frame consists of:
   a- Equivalent columns.
   b- Equivalent slab beams.

Procedure of E.F.M.

1. Equivalent slab beams.
a- Find \( \frac{c_{1A}}{L_1}, \frac{c_{1B}}{L_1} \)
   Where :-
   \( c_{1A} \) and \( c_{1B} \) (width of support parallel to \( L_1 \))
   Go to table (A13-A,B)
   Find :-
   1- \( k_{AB}, k_{BA} \) (stiffness factor)
   \[ K_{AB} = k_{AB} \cdot \frac{E I_{s,b}}{L_1} \]
2. Carry over factor .
\[ C.O.F_{AB} = C.O.F_{BA} \]

3. \( M_{AB} = M_{BA} \) (factor of F.E.M)
\[ F.E.M = M_{AB} \times W_u L_2 L_1^2 \]

2. Equivalent columns .
\[ \frac{c_{1A}}{L_1} \quad \text{where} \quad c_{1A} = T_{equ}. \]

\( k_{AB}, k_{BA} \) (stiffness factor )

\[ C.O.F = 0.5 \]

Torsional member
\[ \frac{1}{K_{ec}} = \frac{1}{\Sigma K_c} + \frac{1}{K_t} \]

Where :-
\( K_{ec} = \text{equivalent columns stiffness}. \)
\( K_c = \text{stiffness column}. \)
\[ K_c = K \times \frac{E_l}{L_1} \]
\[ I_c = \frac{c_2 c_3^2}{12} \]

\( K_t = \text{torsional stiffness of transverse beam}. \)
\[ K_t = \sum \frac{9E_c c}{L_t \left(1 - c_2/L_t\right)^3} \times \frac{I_{s,b}}{I_s} \]

\[ c = \sum \left[ (1 - 0.63) \frac{x}{y} \right] \frac{x^3 y}{3} \]

\( L_t \) = for end frame = \( L_2 \)
\( L_t \) = for interior frame = \( L_2 + L_2^* \)
EX(1): Data :-

1- Three story building  
2- All beam =0.3×0.6 m  
3- All columns =0.3×0.3 m  
4- Slab thickness =0.2 m  
5- Story height =3m  
6- D.L=10 $kn/m^2$ (total)  
7- L.L=5 $kn/m^2$  
8- $f_c=25$ mpa , $f_y=400$ mpa

Required :-

Find all negative moment at interior of x-direction .

Sol:-

Equivalent slab beam section :

For ext. frame (AB)

\[
\frac{c_{1A}}{L_1} = \frac{c_{1B}}{L_1} = \frac{0.3}{5} = 0.06
\]

From table (A13.A)

\[
M_{AB} = M_{BA} = 0.0842
\]

\[
k_{AB}, k_{BA} = 4.08
\]

\[
C.O.F_{AB} = C.O.F_{BA} = 0.505
\]

\[
y = \frac{0.3 \times 0.4 \times 0.4 + 5 \times 0.2 \times 0.1}{0.3 \times 0.4 + 5 \times 0.2} = 0.132 m
\]

\[
I_{s,b} = \frac{5 \times 0.132^3}{3} + \frac{5 \times (0.2 - 0.132)^3}{3} + \frac{0.3 \times 0.4^3}{12} + 0.3 \times 0.4 \times (0.4 - 0.132)^2 = 0.01457 m^4 = 1.46 \times 10^{10} mm
\]

\[
I_{s,b} = \frac{bh^3}{12}
\]

\[
1.46 \times 10^{10} = \frac{3000 \times h^3}{12}
\]

\[
h_{equ.} = 327 mm
\]

\[
E = 4700\sqrt{f_c} = 4700\sqrt{25} = 23500 \frac{n}{mm^2} = 2.35 \times 10^7 kn/m^2
\]

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\[ K_{AB} = K_{BA} = 4.08 \times \frac{E \times I_{s,b}}{L_1} = 4.08 \times \frac{2.35 \times 10^7 \times 0.0146}{5} = 279970 \text{ kn/m} \]

\[ F.E.M = M_{AB} \times W_u L_2 I_1^2 = 0.0842 \times 20 \times 5 \times 5^2 = 210.5 \text{ kn.m} \]

**For int. frame (BC):**

\[ \frac{c_{1A}}{L_1} = \frac{c_{1B}}{L_1} = \frac{0.3}{7} = 0.043 \]

\[ K_{BC} = K_{CB} = 4.04 \times \frac{2.35 \times 10^7 \times 0.0146}{7} = 198018 \frac{\text{kn}}{m} \]

\[ F.E.M = M_{AB} \times W_u L_2 I_1^2 = 0.084 \times 20 \times 5 \times 7^2 = 411.6 \text{ kn.m} \]

**Eq. columns :**

\[ \frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_t} \]

**Column (aA):**

\[ \frac{c_{1A}}{L_1} = \frac{T_{equ}}{L_1} = \frac{0.327}{3} = 0.109 \approx 0.11 \]

\[ k = 7.63 \]

\[ l_c = \frac{c_2 c_1^3}{12} = \frac{0.3 \times 0.3^3}{12} = 6.75 \times 10^{-4} \text{ m}^4 \]

\[ K_c = K \times \frac{E l_c}{L_1} = 7.63 \times \frac{2.35 \times 10^7 \times 6.75 \times 10^{-4}}{3} = 40343.625 \frac{\text{kn}}{m} \]

\[ K_t = \sum \frac{9E_c c}{L_t (1 - c_2/L_t)^3} \times \frac{I_{s,b}}{I_s} \]

\[ c = \sum [(1 - 0.63 \frac{x}{y}) x^3 y] \]

\[ = (1 - 0.63 \frac{0.3}{0.6}) \frac{0.3^3 \times 0.6}{3} + (1 - 0.63 \frac{0.2}{0.4}) \frac{0.2^3 \times 0.4}{3} = 4.43 \times 10^{-3} \]

\[ l_s = \frac{b h^3}{12} = \frac{5 \times 0.2^3}{12} = 3.33 \times 10^{-3} \]

\[ K_t = \frac{9 \times 2.35 \times 10^7 \times 3.33 \times 10^{-3}}{5(1 - 0.3/5)^3} \times \frac{0.0146}{3.33 \times 10^{-3}} = 988177 \frac{\text{kn}}{m} \]

\[ \frac{1}{K_{ec}} = \frac{1}{2 \times 40343.625} + \frac{1}{2 \times 988177} \]
$K_{ec} = 77522.3 \frac{\text{kn}}{\text{m}}$

**Column (bB):**

\[ \frac{c_1 a}{L_1} = 0.11 \]

$K_c = 40343.625$

$c_1 = 5.16 \times 10^{-3}$

$c_2 = 4.49 \times 10^{-3}$

\[ K_t = \frac{9 \times 2.35 \times 10^7 \times 5.16 \times 10^{-3}}{5(1 - 0.3/5)^3} \times \frac{0.0146}{3.33 \times 10^{-3}} = 1152166 \frac{\text{kn}}{\text{m}} \]

$K_{ec} = 77957.5 \frac{\text{kn}}{\text{m}}$

---

<table>
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<tr>
<th>Joint</th>
<th>a</th>
<th>A</th>
<th>B</th>
<th>C</th>
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3- Yield Line Theory of Slab Analysis:

Hint analysis:-

It's known that concrete does not respond elastically to loads of more than about one half the ultimate loads therefore there is a certain in consistency in design of reinforced concrete cross-section by ultimate strength method in to account in elastic consideration while design moment have been found by elastic analysis. This procedure (elastic analysis coupled with in elastic design of section) is not consistent but it's present acceptable due to safety and connectivity.

For astatically indeterminate structure (beam, slab, frame) failure will not occur when the ultimate moment capacity at just one critical cross-section is reached. After formation of plastic hinges at more highly stressed sections, substantial changes may occur in the ratio of moments at critical section as loads are further increased before collapse of the structure take place.

1- elastic:

\[ M_e = \frac{WL^2}{12} \]

\[ W_e = \frac{12M_e}{L^2} \]

2- plastic:

The third hinge forms when:

\[ M^+ = \frac{WL^2}{8} - M_n \]

Plastic analysis

\[ M^+ = M^- \]
\[ \frac{WL^2}{8} - M_n = M_n \]
\[ \frac{WL^2}{8} = 2M_n \]

\[ W_p = \frac{16M_n}{L^2} \]
\[ \frac{W_p}{W_e} = \frac{16}{12} = 1.33 \]

Increment percentage \( \frac{W_p}{W_e} = 33\% \)

- Plastic hinge and collapse mechanism:

\[ M_y = A_s f_y \left( d - \frac{kd}{3} \right) \]

\[ a = BC = 0.85 C \]

\[ M_n = A_s f_y \left( d - \frac{a}{2} \right) \]

\[ M_n > M_y \]
due to increase in lever arm some increase in $M_n$ is due to strain hardening of reinforcement.

After the calculation of resisting moment "$M_n$" is reached. continued plastic rotation is assumed to occur with no change in the applied moment the beam behaviors as if there where a hinge at that points, this hinge differs from the free fraction hinge. If such a plastic hinge forms in a determinate structure will collapse, the resulting system is referred to as a mechanism.

**EX(1):** Find the max. load that can be applied at simply supported beam shown:

**Sol:-**

External work = internal work

$$\sum W\delta = \sum M\phi$$

$$P \cdot \delta = M_n \cdot 2\phi$$

$$P_{p,1} = \frac{2M_n \cdot \frac{\delta}{L/2}}{\delta} = \frac{4M_n}{L} = \frac{4M_{el}}{L}$$

$$P_{pl} = P_{el} \quad \text{Simply Supported}$$

**EX(2):** find the max. load that can be applied at un determinate beam shown:

**Sol:-**

$$\sum W\delta = \sum M\phi$$

$$W\left(\frac{L}{2} \cdot \frac{\delta}{2}\right) \cdot 2 = M_n \cdot 4\phi$$

$$\frac{WL\delta}{2} = M_n \cdot \frac{4 \cdot \delta \cdot 2}{L}$$

$$W_{p,1} = \frac{16M_n}{L^2} > W_{el} = \frac{12M_n}{L^2}$$

**H.W:** find the max. load that can be applied at un determinate beam shown:

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Sol:-

\[ \sum W\delta = \sum M\varphi \]

- for right part:

\[ 3W \cdot 1 = M_n \cdot 4 \cdot \frac{1}{4} \]

\[ W = \frac{M_n}{3} \]

- for left part:

\[ 2W \cdot 1 = M_n \cdot 4 \varphi_2 \]

\[ 2W = M_n \cdot 4 \cdot \frac{1}{5} \]

\[ W = 0.4M_n \]

- for two part:

\[ 3W \cdot 1 + 2W \cdot 1 = M_n \cdot 4\varphi_1 + M_n \cdot 4\varphi_2 \]

\[ W = 0.36M_n \]

**Yield Line Analysis of Slabs:**

The yield line theory is a method for the limit analysis of R.C slabs. The theory first developed by K.W Johansen (Denmark) at 1943. It permits the analysis of irregular as well as regular slabs of a variety of support and loading condition:-
Sol:-

\[ \Sigma W\delta = \Sigma M\varphi \]

\[ W \cdot b \cdot (L - xL) \cdot \delta_1 + W \cdot b \cdot xL \cdot \delta_2 = M_n \cdot b \cdot \varphi_1 + M_n \cdot b \cdot (\varphi_1 + \varphi_2) \]

\[ Wb(L - xL) \cdot \delta_1 + WbxL \cdot \delta_2 = M_n b \cdot \frac{1}{(L-xL)} + M_n b \left( \frac{1}{(L-xL)} + \frac{1}{xL} \right) \]

\[ \frac{Wb}{2} - \frac{Wb}{2} + \frac{Wb}{2} = \frac{Mb}{L - xL} + \frac{Mb}{L - xL} + \frac{Mb}{xL} \]

\[ \frac{WbL}{2} = \frac{2Mb}{L - xL} + \frac{Mb}{xL} \]

\[ W_n = \frac{4M}{L^2(L - x)} + \frac{2M}{xL^2} = \frac{2M}{L^2} \left( \frac{2}{(1 - x)} + \frac{1}{x} \right) \]

\[ \frac{dw}{dx} = 0 \]

\[ \frac{2M_n}{L^2} \left( \frac{2}{(1 - x)^2} + \frac{-1}{x^2} \right) = 0 \]

\[ \frac{2}{(1 - x)^2} = \frac{1}{x^2} \]

\[ x^2 + 2x - 1 = 0 \quad \text{solve the eq. get} \]

\[ x_1 = 0.414 \]

\[ x_2 = -2.414 \]
\[ W_n = \frac{2M_n}{L^2} \left( \frac{2}{(1 - 0.414)} + \frac{1}{0.414} \right) \]

\[ W_n = \frac{11.657 M_n}{L^2} \]

**H.W:** A continuous one way slab is uniformly loaded and has an ultimate moment capacity of 24 kn.m/m at "A" 30 kn.m/m at "B" and 18 kn.m/m at "C" using the principle of virtual work find the collapse load \( W_n \)?

**Hint:** \( W_n = 9.99 \text{ kn/m}^2 \)

**Sol:**

\[ \sum W \delta = WE \]

\[ = W \times x \times 2 \times \frac{1}{2} + W \times (6 - x) \times 2 \times \frac{1}{2} \]

\[ = Wx + 6W - Wx = 6W \]

\[ \sum M \phi = WI \]

\[ = 24 \times 2 \times \frac{1}{x} + 18 \times 2 \times \frac{1}{x} + 18 \times 2 \times \frac{1}{6-x} + 30 \times 2 \times \frac{1}{6-x} \]

\[ = \frac{48}{x} + \frac{36}{x} + \frac{36}{6-x} + \frac{60}{6-x} = \frac{84}{x} + \frac{96}{6-x} \]

\[ \sum W \delta = \sum M \phi \]

\[ 6W = \frac{84}{x} + \frac{96}{6-x} \]

\[ W = \frac{14}{x} + \frac{16}{6-x} \]

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\[ Sh. Rasheed \]
\[
\begin{align*}
\frac{dw}{dx} &= 0 \\
-14 \frac{x}{x^2} + 16 \frac{1}{(6-x)^2} &= 0 \\
504 - 168x - 2x^2 &= 0 & \text{solve the eq. get} \\
x &= 2.89988 \text{m} & \text{o.k} \\
W &= \frac{14}{2.89} + \frac{16}{6 - 2.89} = 9.98 \text{ kn/m}^2
\end{align*}
\]

**Predication of yielding patterns for two way slab:**

1. Yield line are generally straight.
2. Axes of rotation pass over any column.
3. Yield line pass through the intersection of the axes of rotation of adjacent slab segment.
4. Axes of rotation generally lies along line of supported.
5. Any symmetry in the slab is maintained in the yield line patterns.

**Notation:**
- Column
- Simply support
- Fixed support
- +ve yield line
- -ve yield line
- Point load
- Axes of rotation

**EX(2):** For the uniformly loaded isotropically reinforced slab shown below determine the collapse load \(W_u\) assuming the plastic moment perimeter \(M_u\)?

**Sol:**
\[
\begin{align*}
\sum W\delta &= \sum M\varphi \\
\sum W\delta &= \left(\left(W \times \frac{L}{2} \times \frac{L}{2}\right) \times \frac{1}{3}\right) \times 4 \\
\sum W\delta &= \frac{W L^2}{3} \times 30
\end{align*}
\]

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Or (using the volume of Pyramid as shown):

\[ \sum W\delta = \frac{1}{3} W * L^2 = \frac{WL^2}{3} \]

\[ \sum M\phi = 4 \left( M_n * L * \frac{1}{L/2} \right) = 8M_n \]

\[ \frac{WL^2}{3} = 8M_n \]

\[ W_n = \frac{24M_n}{L^2} \]

**H.W:-**

**Sol:-**

\[ \tan 30 = \frac{a}{5} \]

\[ a = 2.88 \text{ m} \]

\[ WE = \left( W * \frac{1}{2} * 10 * \frac{5\sqrt{3}}{3} * \frac{1}{3} \right) * 3 \]

\[ \delta = \frac{1}{3} \]

\[ WE = 14.434 \ W \]

\[ WI = \left( M_n * 10 * \frac{1}{5\sqrt{3}/3} \right) * 3 = 10.3923M_n \]

\[ 14.434W = 10.3923M_n \]

\[ W = 0.72 \ M_n \]

Or

\[ \sum M\phi = 3M_n \left( a \left( \frac{1}{b} + \frac{1}{b} \right) \right) = 10.4M_n \]

\[ \sum W\delta = \sum M\phi \]
EX(3): Determine the collapse load $W_n$ for the uniformly loaded slab shown below, assuming the plastic moment of resistance / meter width = $M_n$?

Sol:-

$$\sum W\delta = \frac{1}{3} W \times \frac{\pi r^2}{2} \times 1$$

$$\sum M\phi = 2 \times \frac{\pi r^2}{2} \times M_n \times \frac{1}{r}$$

$$\sum M\phi = \sum W\delta$$

$$W = \frac{6M_n}{r^2}$$

EX(4): Determine the collapse load $W_n$ for the uniformly loaded slab shown below, assuming the plastic moment of resistance / meter width = $M_n$?

Sol:- mode(1)

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\[ \sum W\delta = WE \]

\[ = W \cdot \frac{1}{2} \cdot L \cdot xL \cdot \frac{1}{3} + \left( W \cdot \frac{L}{2} \cdot \left( L - xL \right) \cdot \frac{1}{2} \right) \cdot 2 \]

\[ + \left( W \cdot \frac{L^2}{2} \cdot \left( 1 - xL \right) \right) \cdot \frac{1}{3} \cdot 2 \]

\[ = \frac{WxL^2}{6} + \frac{WL^2}{2} - \frac{WxL^2}{2} + \frac{WxL^2}{6} = -\frac{WxL^2}{6} + \frac{WL^2}{2} = \frac{WL}{6} (3 - x) \]

Or (using the volume of Pyramid as shown):

\[ \sum W\delta = W \left( \frac{1}{3} \cdot \frac{xL \cdot L}{2} \right) \cdot 1 + \frac{1}{2} \cdot \left( (1 - x) \cdot L \right) \cdot L \cdot 1 \]

\[ = \frac{WL^2}{6} (3 - x) \]

\[ \sum M\phi = M_n \left( L \cdot \frac{1}{xL} \right) + 2 \left( M_n \cdot L \cdot \frac{1}{L/2} \right) = M_n \left( \frac{1}{x} + 4 \right) \]

\[ \sum M\phi = \sum W\delta \]

\[ \frac{WL^2}{6} (3 - x) = M_n \left( \frac{1}{x} + 4 \right) \]

\[ W_n = \frac{6M_n}{L^2} \cdot \frac{(4x + 1)}{(3x - x^2)} \]

\[ \frac{dw}{dx} = 0 \]

\[ \frac{6M_n}{L^2} \cdot \frac{(3x - x^2) \cdot 4 - (4x + 1)(3 - 2x)}{(3x - 2x^2)^2} = 0 \]

\[ 4x^2 + 2x - 3 = 0 \] solve the equation get:

\[ x = 0.65 \text{ m} > 1 \text{ t} \text{e mode o.k} \]
\[ W_n = \frac{6M_n}{L^2} \times \frac{(4 \times 0.65 + 1)}{(3 \times 0.65 - 0.65^2)} = 14.14 \frac{M_n}{L^2} \]

Sol:- mode(2):

\[ WE = \left( W \times \frac{1}{2} \times xL \times L \times \frac{1}{3} \right) \times 4 + W(L - 2xL) \times L \times \frac{1}{2} \]

\[ = \frac{2WxL^2}{3} + \frac{WL^2}{2} - WXL^2 = WL^2 \left( \frac{1}{2} - \frac{x}{3} \right) \]

\[ WI = 2M_n \times L \times \frac{1}{xL} + M_n \times 2xL \times \frac{1}{L} \]

\[ = \frac{2M_n}{x} + 2xM_n = 2M_n \left( \frac{1}{x} + x \right) \]

\[ W = \frac{12M_n}{L^2(3 - 2x)} \times \frac{(1 + x^2)}{x} = 12M_n \left( \frac{1 + x^2}{3x - 2x^2} \right) \]

\[ \frac{dw}{dx} = 0 \]

\[ \frac{12M_n}{L^2} \left( \frac{(3x - 2x^2) \times 2x - (1 + x^2) \times (3 - 4x)}{(3x - 2x^2)^2} \right) = 0 \]

\[ 3x^2 + 4x - 3 = 0 \]

\[ x = 0.535 < \frac{L}{2} \]

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EX(5): Determine the collapse load \( W_n \) for the uniformly loaded slab shown below, assuming the plastic moment of resistance / meter width = \( M_n \)?

\[\Sigma W\delta = WE = \left( W \times \frac{L \times x \times L}{2} \right) + W \times (2L - 2xL) \times L \times \frac{1}{2} \]

\[= \frac{WL^2}{3} \times (3 - x)\]

\[\Sigma M\phi = WI = M_n \times 2L \times \frac{1}{xL} + M_n \times 2L \times \frac{1}{L} + M_n \times 2xL \times \frac{1}{L} \]

\[= \frac{2M_n}{x} + 2M_n + 2M_n x = 2M_n \left( \frac{1}{x} + 1 + x \right)\]

\[W_n = \frac{M_n}{L^2} \left( \frac{x^2 + x + 1}{3x - x^2} \right) = 8.14 \times \frac{M_n}{L^2}\]

EX(6): For the uniformly loaded isotropic ally reinforced slab shown below; determine the collapse load \( W_n \)? Assume the plastic moment of resistance per meter width of slab for positive and negative section is \( 6 \frac{kn.m}{m} \)?
Sol :

\[ \Sigma W \delta = W \frac{x \times 6}{2} \times \frac{1}{3} + 2W \frac{(2.4-0.6x) \times 5}{2} \times \frac{1}{3} = 4W \]

\[ \Sigma M \phi = M_n \left( 2 \times 5 \times \frac{1}{a} + (6+6) \times \frac{1}{x} \right) = M_n \left( \frac{12}{x} + \frac{10}{a} \right) \]

\[ = M_n \left( \frac{12}{x} + \frac{10}{2.4-0.6x} \right) \]

\[ \Sigma W \delta = \Sigma M \phi \]

\[ 4W = M_n \left( \frac{12}{x} + \frac{10}{2.4-0.6x} \right) \]

\[ W = \frac{M_n}{4} \left( \frac{12}{x} + \frac{10}{2.4-0.6x} \right) \]

\[ \frac{dw}{dx} = 0 \]

\[ x = 1.836 \text{ M}_n \]

\[ \therefore W_n = 21.36 \text{ kn} \]

\[ \text{EX(7): Determine the collapse load } W_n \text{ for the uniformly loaded slab shown below ,assuming the plastic moment of resistance / meter width } = M_n ? \]

Sol: -

\[ WE = \Sigma W \delta \]

\[ = 2W \frac{2L}{2} \times \frac{1}{3} + 2W \frac{1.73L(x - L)}{2} \times \frac{1}{3} + W \times (40 - 2x) \times 1.73L \times \frac{1}{2} \]

\[ WI = M_n \left( 2 \times (2L + (2L - b)) + (2L + 2(x - L)) \right) \]

\[ WI = 2M_n (3.5L + 2x) \]

\[ W = 3.275 \frac{M_n}{L^2} \]

\[ a = L \times \sin 60 \]
b = L \cos 60°

**EX(8):** Determine the collapse load $W_n$ for the uniformly loaded slab shown below, assuming the plastic moment of resistance per meter width = $M_n$.

\[
\frac{\sqrt{3}}{2} = \frac{a}{2L}
\]

\[a = \sqrt{3}L\]

**Sol:**

\[
WE = \sum W\delta
\]

\[
= 2W \left( \frac{2L \times 2\sqrt{3}L}{2} \times \frac{2}{3} - \left( \frac{L \times \sqrt{3}L}{2} \times \frac{1}{3} \right) \right) = 7W \frac{L^2}{\sqrt{3}}
\]

Or (using the volume of Pyramid as shown):

\[
WE = \frac{1}{3} \left( \frac{4L \times 2}{2} \times 2\sqrt{3}L - \frac{2L \times 1}{2} \times \sqrt{3}L \right) = 7W \frac{L^2}{\sqrt{3}}
\]

\[
\sum M_\varphi = 2M_n \left( 2L \times \frac{2}{\sqrt{3}L} + 3L \times \frac{2}{\sqrt{3}L} - 1.5L \times \frac{2}{\sqrt{3}L} \right) = \frac{14M_n}{\sqrt{3}}
\]

\[
\sum W\delta = \sum M_\varphi
\]

*Dr. Laith  
Sh. Rasheed*
Isotropically reinforced slabs:

Slabs reinforced identically orthogonal direction or the tropically reinforced slabs. Slabs with different reinforced ratio in orthogonal direction.

For isotropically reinforced slabs the moment at any direction $\alpha$ may be shown to be equal to either of the moment in the two orthogonal direction.

Prove:

$$M_xcb = M_xcd \cos \alpha + M_y \cos (90 - \alpha)$$

$$M_x = M_x \frac{cd}{cb} \cos \alpha + M_y \frac{ac}{cb} \cos (90 - \alpha)$$

$$= M_x \cos^2 \alpha + M_y \cos^2 (90 - \alpha)$$

$$M_x = M_x \cos^2 \alpha + M_y \sin^2 \alpha$$

Since $M_x = M_y = M$ (isotropic reinforcement)

$$\therefore M_x = M_x \cos^2 \alpha + \sin^2 \alpha$$

$$\therefore M_x = M = M_x = M_y$$

$$\therefore M_x = M_y$$ are moment / unit width of slab

$M_x$: are moment / unit width of slab
4- Prestress Concrete :

Prestressed concrete is a particular reinforced concrete. Prestressing involves the application of an initial compressive load on a structure to reduce or eliminate the internal tensile forces and thereby control or eliminate cracking. The initial compressive load is imposed and sustained by highly tensioned steel reinforcement reacting on the concrete.

A prestressed section is considerably stiffer than the usually cracked reinforced section.

Prestressing may also impose internal forces which are of opposite sign to the external load and may therefore significantly reduce or even eliminate deflection.

Advantage of prestressing:

1- Smaller sections will be required for design (the entire section remain effective for stress).
2- Large span will be possible due to the weight reduction.
3- Deflection under working load will be reduced (Cambering).
4- Prestress reduces diagonal tension stresses at working load. This load to use modified "I" and "T" section.

Type of Prestressed Concrete:

1- Pretensioned Concrete:

The Prestressing tendons are initially tensioned between fixed abutments and anchored with the from work in place, the concrete is cast around the highly stressed steel tendons and cured. When the concrete has reached its required strength the wires are otherwise released from abutment. As the highly stressed steel attempts to contract the concrete is compressed. Prestress is imparted via bond between the steel and concrete, to decrease the construction cycle time, Steam curing may be employed, to facilitate rapid
concrete strength gain and the concrete is often stressed with 24 hrs. of casting.

2- Post-tensioned Concrete:

The concrete is cast around hollow ducts which are fixed to any desired profile the steel tendons are usually in place, unstressed in the ducts during the concrete pour, or alternatively may be threaded through the ducts at some later time when the concrete has reached its required strength, the tendons are tensioned. Tendons may be stressed from one end with the other end, the tendons are then anchored at each stressing ends. After the tendons have been anchored and no further stressing the tendons are often filled with grout under pressure. In this way the tendons are bonded to the concrete and are more efficient in controlling cracking and providing ultimate strength.
- **Prestress Losses** :-
  1- immediate losses :
     a- bend slip .
     b- fraction .
     c- elastic shortening .
     d- shrinkage .
  2- long term losses:
     a- creep in concrete .
     b- relaxation in steel .

- **Prestress Forces** :
  \( P_j \) = jacking force
  \( P_i \) = immediat prestress force \((P_j - \text{immediat losses})\)
  \( P_e \) = effective prestress force \((P_j - \text{long term losses})\)
Method of Analysis:

1- working stress method.

This analysis is done in two stages of loading:

a- initial loading stage where the stress are calculated after complete of immediate losses.

b- service loading stage, where the stress are calculated after all losses.

a- initial loading stage:

\[
\sigma_t = \frac{p_i}{A} + \frac{p_i \cdot e \cdot c_t}{l} - \frac{M_g \cdot c_t}{I} \\
\sigma_b = \frac{-p_i}{A} + \frac{p_i \cdot e \cdot c_b}{l} + \frac{M_g \cdot c_b}{I}
\]

\[
r = \sqrt{\frac{1}{A}} \quad \leftrightarrow \quad r^2 = \frac{1}{A} \quad ; \quad S = \frac{1}{c}
\]

\[
\therefore \sigma_t = \frac{-p_i}{A} \left(1 - \frac{e_c t}{r^2}\right) - \frac{M_g}{S_t}
\]

\[
\therefore \sigma_b = \frac{-p_i}{A} \left(1 + \frac{e c_b}{r^2}\right) + \frac{M_g}{S_b}
\]

Stress at end span of beam:

\[
\sigma_t = \frac{-p_i}{A} \left(1 - \frac{e c_t}{r^2}\right)
\]
\[ \sigma_b = \frac{-p_i}{A} \left(1 + \frac{ec_b}{r^2}\right) \]

**b-service load stage:**

\[ \sigma_t = \frac{p_e}{A} \]
\[ \sigma_t = \frac{p_e \cdot e \cdot c_t}{l} \]
\[ \sigma_t = \frac{M_g \cdot c_t}{l} \]
\[ \sigma_t = \frac{M_s \cdot c_t}{l} \]

Stress at mid span of beam:

\[ \sigma_b = \frac{-p_e}{A} \left(1 - \frac{ec_t}{r^2}\right) - \frac{M_g}{S_t} - \frac{M_s}{S_t} \]

Stress at end of span:

\[ \sigma_b = \frac{-p_e}{A} \left(1 + \frac{ec_b}{r^2}\right) + \frac{M_g}{S_b} + \frac{M_s}{S_b} \]

The permissible stresses according ACI -318 code (18-4):

1- initial loading stage:
   - compression stress \( \leq 0.6 \cdot f_{c,i} \) (at mid and end)
Where:

\( f_{ci} \) = initial strength at yang ages.

- tensile stress at middle \( \leq 0.25 \sqrt{f_{ci}} \)
- tensile stress at end \( \leq 0.5 \sqrt{f_{ci}} \)

2- Service loading stage:
- compressive stress due to sustained load \( \leq 0.45 f_c \)
- compressive stress due to total load \( \leq 0.6 f_c \)
- tensile stress \( \leq 0.5 \sqrt{f_c} \)

**EX(1):** For the beam shown check the stresses according to ACI code?

assuming:

- immediate losses = 10%
- total losses = 20%
- add. D.L = 20 kN/m²
- L.L = 10 kN/m² with 10% sustained.
- \( f_c = 21 \text{ mpa} \), \( f_{ci} = 15 \text{ mpa} \)
- jacking prestress = 0.8 \( f_{py} \)

**Sol:**

\[
A = 300 \times 900 = 27 \times 10^4 \text{mm}^2
\]

\[
I = \frac{300 \times 900^3}{12} = 1.8225 \times 10^{10} \text{mm}^4
\]

\[
r^2 = \frac{I}{A} = 67500 \text{mm}^2
\]

\[
S_t = S_b = \frac{I}{c} = 4.05 \times 10^7
\]

1- initial loading stage:
University of Kerbala  
Civil Department  
Engineering College  
Reinforced concrete design II  

\[ \sigma_t = \frac{-p_i}{A} \left( 1 - \frac{ec_t}{r^2} \right) \ldots \ldots (1) \]

\[ \sigma_b = \frac{-p_i}{A} \left( 1 + \frac{ec_b}{r^2} \right) \ldots \ldots (2) \]

\[ P_j = 0.8 \times 1750 \times 5 \times 92.9 = 650.3 \text{ kn} \]

\[ P_i = 650.3 - 0.1 \times 650.3 = 585.27 \text{ kn} \]

\[ \sigma_t = \frac{-585.27 \times 10^3}{27 \times 10^4} \left( 1 - \frac{400 \times 450}{67500} \right) = +3.63 \text{ mpa} \leq \frac{\sqrt{f_{ci}^2}}{2} = \frac{\sqrt{15}}{2} \]

\[ = 1.9 \text{ not o.k} \]

\[ \sigma_b = \frac{-585.27 \times 10^3}{27 \times 10^4} \left( 1 + \frac{400 \times 450}{67500} \right) = -7.96 \text{ mpa} \leq 0.6 \sqrt{f_{ci}^2} = 0.6 \sqrt{15} \]

\[ = 9 \text{ o.k} \]

- at mid span:

\[ \sigma_t = \frac{-p_i}{A} \left( 1 - \frac{ec_t}{r^2} \right) - \frac{M_g}{S_t} \]

\[ M_g = \frac{WL^2}{8} \]

\[ W_g = 0.3 \times 0.9 \times 24 = 6.48 \text{ kn/m}^2 \]

\[ \therefore M_g = \frac{6.48 \times 20^2}{8} = 324 \text{ kn.m} \]

\[ \sigma_t = -3.63 - \frac{324 \times 10^6}{4.04 \times 10^7} = -4.39 \leq 9 \text{ o.k} \]

\[ \sigma_b = \frac{-p_i}{A} \left( 1 + \frac{ec_b}{r^2} \right) + \frac{M_g}{S_b} = -7.69 + 8.02 = 0.06 \text{ mpa} \leq \frac{\sqrt{f_{ci}^2}}{4} = \frac{\sqrt{15}}{4} \]

\[ = 0.95 \text{ o.k} \]

2- Service loading stage:
- at end sec. (for sustained and total)
\[ \sigma_t = \frac{-p_e}{A} \left( 1 - \frac{e_c}{r^2} \right) = \frac{-520.24 \times 10^3}{27 \times 10^4} \left( 1 - \frac{400 \times 450}{67500} \right) = 3.22 \text{ mpa} \]
\[ \leq 0.5\sqrt{f_c} = 0.5\sqrt{21} = 2.29 \text{ not o.k} \]

\[ \sigma_b = \frac{-p_e}{A} \left( 1 + \frac{e_c}{r^2} \right) = \frac{-520.24 \times 10^3}{27 \times 10^4} \left( 1 + \frac{400 \times 450}{67500} \right) = -7.08 \text{ mpa} \]
\[ \leq 0.45f_c = 9.45 \text{ o.k} \]

- at mid span (for sustained loading):

\[ \sigma_t = \frac{-p_e}{A} \left( 1 - \frac{e_c}{r^2} \right) - \frac{M_g}{S_t} - \frac{M_s}{S_t} \]

\[ W_s = 20 + 0.1 \times 10 = 21 \]

\[ M_s = \frac{Wl^2}{12} = \frac{21 \times 20^2}{12} = 1050 \]

\[ \therefore \frac{M_s}{S_t} = \frac{1050 \times 10^6}{4.04 \times 10^7} = 26 \]

\[ \sigma_t = 3.22 - 8.02 - 26 = -30.6 \text{ mpa} \leq 0.45f_c = 9.45 \text{ not o.k} \]

\[ \sigma_b = -7.08 + 8.02 + 26 = 26.9 \text{ mpa} \leq 0.5\sqrt{f_c} = 0.5\sqrt{21} = 2.29 \text{ not o.k} \]

- at mid span (for total loading):

\[ M_s = \frac{(10 + 20) \times 20^2}{8} = 1500 \text{ kn.m} \]

\[ \therefore \frac{M_s}{S_t} = \frac{1500 \times 10^6}{4.04 \times 10^7} = 37.13 \text{ mpa} \]

\[ \sigma_t = \frac{-p_e}{A} \left( 1 - \frac{e_c}{r^2} \right) - \frac{M_g}{S_t} - \frac{M_s}{S_t} = 3.22 - 8.02 - 37.13 = -41.93 \text{ mpa} \]
\[ \leq 0.6 \times 21 = 12.6 \text{ not o.k} \]

\[ \sigma_b = \frac{-p_e}{A} \left( 1 + \frac{e_c}{r^2} \right) + \frac{M_g}{S_b} + \frac{M_s}{S_b} = -7.08 + 8.02 + 37.13 = 38.07 \text{ mpa} \]
\[ \leq 0.5 \times 21 = 2.29 \text{ not o.k} \]
- rupture cracking :

\[ f_t \leq 0.62 \sqrt{f_c} \]

Where:

\[ f_t = \text{tensile stress} \]

### 2- Ultimate Strength Method :

For member with bonded tendons:

(pretension prestress member)

\[
\psi_{ps} = \psi_{pu} \left[ 1 - \frac{\gamma_p}{\beta_1} \left( \rho_p \frac{f_{pu}}{f_c} + \frac{d}{d_p} (\omega - \omega) \right) \right]
\]

If compression steel taken into account then:

a. \( \rho_p \frac{f_{pu}}{f_c} + \frac{d}{d_p} (\omega - \omega) \geq 0.17 \)

b. \( d \leq 0.15 \ d_p \)

where:

\[ \gamma_p = 0.55 \] for \( \frac{f_{py}}{f_{pu}} \geq 0.8 \) (typical high strength bars)

\[ \gamma_p = 0.4 \] for \( \frac{f_{py}}{f_{pu}} \geq 0.85 \) (typical ordinary strand)

\[ \gamma_p = 0.28 \] for \( \frac{f_{py}}{f_{pu}} \geq 0.9 \) (typical low relaxation strand)

\[ \beta_1 = 0.85 \] if \( 17 \leq f_c \leq 28 \)

\[ \beta_1 = \left[ 0.85 - \frac{0.05}{7} (f_c - 28) \right] \geq 0.65 \]

\[ \omega = \rho \frac{f_y}{f_c}; \quad \omega' = \rho' \frac{f_y}{f_c}; \quad \omega_p = \rho_p \frac{f_{ps}}{f_c} \]

\[ \rho = \frac{A_s}{bd}; \quad \rho' = \frac{A_s}{bd}; \quad \rho_p = \frac{A_{ps}}{bd_p} \]

For member with un bonded tendons:
(post tension prestress members)

a- if \( \frac{\text{span}}{\text{depth}} \leq 35 \) then:

\[
\begin{align*}
\psi_{\text{ps}} &= f_{\text{se}} + 70 + \frac{f_c}{100p_p} \\
\psi_{\text{ps}} &\leq f_{\text{py}} \\
\psi_{\text{ps}} &\leq f_{\text{se}} + 420
\end{align*}
\]

\text{Use min.}

\[
\psi_{\text{ps}} 
\]

b- if \( \frac{\text{span}}{\text{depth}} > 35 \) then:

\[
\begin{align*}
\psi_{\text{ps}} &= f_{\text{se}} + 70 + \frac{f_c}{100p_p} \\
\psi_{\text{ps}} &\leq f_{\text{py}} \\
\psi_{\text{ps}} &\leq f_{\text{se}} + 210
\end{align*}
\]

\text{Use min.}

\[
\psi_{\text{ps}} 
\]

\[
f_{\text{se}} \geq 0.5 f_{\text{pu}}
\]

Strength of prestress section:

Then nominal flexural strength of prestress beam:

1- for rectangular section:

\[
\text{N.A}
\]

\[
M_n = \psi_{\text{ps}} A_{\text{ps}} \left( d_p - \frac{a}{2} \right) \quad \ldots \ldots \quad (1)
\]

Or

\[
M_n = 0.85 f_c a b \left( d_p - \frac{a}{2} \right) \quad \ldots \ldots \quad (2)
\]

\[
a = \frac{A_{\text{ps}} \psi_{\text{ps}}}{0.85 f_c b}
\]
If section "T": 

a- if stress block at with in flange depth (Rectangular section): 

\[ b - \text{stress block is over the flange depth (T-sect.)} \]
For partially prestressed
\[ T_w = A_{psf} \cdot f_{ps} + A_s \cdot f_y - T_f \]
\[ T_f = C_f = 0.85 f_c (b_f - b_w) \cdot h_f \]

**Prestress reinforcement limits according ACI -318 code.**

1- the reinforcement ratio must be .
   a- \( \omega_p \leq 0.36 \beta_1 \)
   b- \( (\omega_p + \frac{d}{d_p} (\omega - \omega')) \leq 0.36 \beta_1 \)
   c- \( A_{smin.} = 0.004 A_t \)

\( A_t = \text{area of tensile zone only.} \)

**Rupture checking of prestress member**
\[ M_R = S_t \left( f_r + \frac{P_e}{A_c} \right) + P_e \cdot e \]
\[ f_r = \text{modulus of rupture} \leq 0.62 \sqrt{f_c} \]

Or
EX(1): A simply supported prestress beam with un bonded tendons for the cross-section shown below find the ultimate moment strength then check the reinforcement limit for the section ? also check the rupture case ?

Assume :-
\[ f_{pu} = 1860 \text{ mpa} \]
\[ f_c = 30 \text{ mpa} \]
\[ P_j = 0.8 f_{pu} \]
\[ \text{losses} = 100 \text{ kn} \]

**Sol:-**

Since un bonded tendons
\[
\frac{\text{span}}{\text{depth}} = \frac{20}{0.7} = 28.57 < 35
\]
\[
f_{ps} = f_{se} + 70 + \frac{f_c}{100 \rho_p} \geq 0.5 f_{pu} = 930
\]
\[
f_{se} = \begin{cases} 
P_j = \text{total losses} = P_e 
\end{cases}
\]
\[
f_{se} = 0.8 \times 1860 - \frac{100 \times 1000}{3 \times 92.9} = 1129.2 > 930 \text{ o.k}
\]
\[
f_{ps} = 1129.2 + 70 + \frac{30}{1000} \left( \frac{3 \times 92.9}{250 \times 650} \right) = 1374 \text{ mpa}
\]
\[
f_{ps} \leq f_{py} \text{ canceled}
\]
\[
f_{ps} = f_{se} + 420 = 1549.2 \text{ mpa}
\]
\[
\therefore f_{ps} = 1347 \text{ mpa}
\]

\[
M_n = 0.85 f_c \cdot a \cdot b \left( d - \frac{a}{2} \right)
\]

\[
a = \frac{A_{ps} f_{ps}}{0.85 f_c \cdot b} = \frac{1347 \cdot 3 \cdot 92.9}{0.85 \cdot 30 \cdot 250} = 60.06 \text{ mm}
\]

\[
M_u = 237 \times 10^6 = 213 \text{ kN.m}
\]

For the reinforcement limit

1- \( \omega_p \leq 0.36 \beta_1 \)

\[
\omega_p = \rho_p \frac{f_{ps}}{f_c} = \frac{3 \cdot 92.9}{250 \cdot 650} \cdot \frac{1374}{30} = 0.07855
\]

\[
\beta_1 = \left[ 0.85 - \frac{0.05}{\gamma} (f_c - 28) \right] = 0.8357
\]

\( \omega_p \leq 0.36 \cdot 0.8357 = 0.3 \text{ o.k} \)

2- \( \omega_p + \frac{d}{d_p} (\omega - \omega') \leq 0.36 \beta_1 \)

3- \( A_{smin.} = 0.004 A_t = 0.004 \times (350 \times 250) = 350 \text{ mm}^2 \)

\[
M_r = S_t \left( f_r + \frac{P_e}{A_c} \right) + P_e \cdot e
\]

\[
S_t = \frac{1}{C_b} = \frac{7.14 \times 10^9}{350} = 20.4 \times 10^6 \text{ mm}^3
\]

\( f_r = 0.62 \sqrt{f_c} = 3.39 \text{ mpa} \)

\( P_e = 1129 \times 3 \times 92.9 \times \frac{1}{1000} = 314.7 \text{ kn} \)

\[
M_r = 20.4 \times 10^6 \times \left( 3.39 + \frac{314.7 \times 10^3}{250 \times 700} \right) + 314.7 \times 10^3 \times 300 = 2 \times 10^8
\]

\( 1.2M_{cr} = 240.3 \text{ kN.m} < 213 \text{ kN.m not o.k} \)
EX(2): A pretensioned S.S double tee-beam have a span (7m) , $\gamma_p =0.85$ , tendons steel area (600$\text{mm}^2$), $b_f = 250\text{mm}$ , $f_{pu}=1860 \text{ mpa}$.

Req.: check if the factored moment capacity are stratifies with ACI-code requirement (assuming $e=149 \text{ mm}$ , $c_t=105 \text{ mm}$ , $c_b=305 \text{ mm}$).

Sol:-

$C=0.85 \ f_{c'} \ b_f \ a$

$T = A_{ps} f_{ps}$

$a = \frac{A_{ps} f_{ps}}{0.85 f_{c'} b_f}$

$f_{ps} = f_{pu} \left[ 1 - \frac{\gamma_p}{\beta_1} \left( \rho_p \frac{f_{pu}}{f_{c'}} + \frac{d}{d_p} (\omega - \phi) \right) \right]$

$\beta_1 = \left[ 0.85 - \frac{0.05}{7} (f_{c'} - 28) \right] = 0.835$

$\rho_p = \frac{A_{ps}}{bd_p} = \frac{600}{2500(149 + 105)} = 0.0009448$

$f_{ps} = 1860 \left[ 1 - \frac{0.85}{0.835} \left( 0.0009448 \times \frac{186}{30} \right) \right] = 1749 \text{ mpa}$

$a = \frac{0.85 \times 30 \times 2500}{600 \times 1749} = 16.48 < 51 \text{ mm o.k (rectangular section)}$

$M_n = f_{ps} \ A_{ps} \left( d_p - \frac{a}{2} \right) = 257.9 \text{ kn.m}$

$M_u = 0.9 M_n = 232.1 \text{ kn.m} > 1.2 M_{cr}$

$M_{cr} = \frac{f_r \ I_g}{\gamma_t} = 31.2$

$232 > 31.2 \text{ o.k}$